RADAR CLUTTER MODELLING AND COHERENT TARGET DETECTION

1. Prerequisites

- Good background in Mathematics, Physics.
- Good background in probability, random variables, and stochastic processes.
- Basic knowledge of statistical decision theory and radar principles.
- Basic knowledge of MATLAB programming.

2. Course format and dates

The course is given in five days over a week period, intensive format, followed by a further three weeks follow up sessions of five hours per week to be organized by using videoconference tools, such as Skype or others.

The student assessment is organized into two tests:

1) solutions of assigned drill problems;
2) three hours examination.

During the intensive five-day course, practical sessions also with the use of MATLAB will be interleaved with classic lectures. Practical sessions are intended to strengthen the understanding of the theory and are based on programming and running routines that implement algorithms that are explained during the lectures. The student will familiarize with the problems and will understand how to set system parameters to achieve desired performances.

Follow up sessions will aim at

1. providing support for solving the assigned drill problems;
2. providing further clarifications about course topics;
3. giving specific seminars on topics related to assigned drill problems.

3. Staff

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4. Course description

The course is organized in three parts, which mainly cover aspects related to radar clutter modelling and analysis, optimum and adaptive radar detection of targets embedded in correlated Gaussian clutter and in heavy-tailed non-Gaussian clutter.
4.1. Radar Clutter Modelling and Analysis

The problem of coherent detection splits naturally into two fundamental sub-problems. In the first sub-problem, a model must be formulated for the underlying disturbance (noise and/or clutter) from which the signals of interest are to be discriminated. The solution of this problem must achieve a balance between the faithful representation of the actual disturbance and the mathematical tractability of the resulting model. In the second sub-problem, a detection structure which minimizes the loss relative to the optimum detection structure while at the same time being easily to implement must be obtained. Often these sub-problems are not solved independently of each other. The mathematical tractability of the disturbance model can be a significant factor in determining the ease of implementation of the resulting detection structure.

The statistical analysis and modelling of radar clutter echoes is therefore a central issue for the design and performance evaluation of radar systems. Aim of this first part is to describe the state-of-the-art approaches to the modelling and understanding of land and sea clutter echoes and their implications on performance prediction and signal processors design. First, we introduce radar sea and ground clutter phenomena, measurements and measurement limitations, at high and low resolution, high and low grazing angles with particular attention to classical model for radar cross section (RCS) prediction. Most part of the lessons will be dedicated to modern statistical and spectral models for high resolution sea and ground clutter and to the methods of experimental validation using recorded data sets. Some preliminaries on the statistic methods will be provided before application on real data.

In this part of the course, the following topics will be covered:

- amplitude statistic modelling of radar clutter, there including phenomenological and empirical distributions;
- spectral modelling of radar clutter, with specific attention to echoes from windblown vegetation and sea surface;
- statistical models for coherent sequences of radar echoes received by multi-parametric radar sensor;
- experimental validation of clutter models on recorded data sets, for both sea and ground clutter, with particular attention to data non-stationarity.

4.2. Optimum and adaptive radar detection of targets in Gaussian clutter

Coherent radar target detection against Gaussian clutter is the subject of the second part of the course. For many years, especially in low resolution radars, clutter echoes were modelled as a Gaussian process. In fact, a radar receiver performs linear averaging at the antenna (remember that
the antenna pattern integrates everything in its pattern), IF bandpass filters, baseband anti-aliasing filters, pulse compression filters, etc. The Central-Limit Theorem (CLT) therefore applies and the output tends to be Gaussian, even when the input is not Gaussian. Optimal and adaptive radar detection algorithms designed to operate against Gaussian clutter have been widely investigated in the radar community. Optimal processing (2-D space-time or 1-D spatial (angle) or 1-D temporal (Doppler) processing) refers to the case of a priori known disturbance covariance matrix and the resulting performance represents a benchmark for any realistic (adaptive) approach. Adaptive processing refers to the case where the disturbance matrix is unknown and must be estimated on-the-fly from the observed data in a coherent processing interval (CPI).

In this part of the course, the following topics related to coherent detection of radar targets in the presence Gaussian clutter will be covered:

- array data model for target signal, clutter, jamming and noise;
- structure and interpretations of the optimum coherent detector in correlated Gaussian clutter: (i) the likelihood ratio test; (ii) the estimator-correlator; (iii) the whitening matched filter;
- suboptimum detectors in Gaussian clutter: the constant false alarm rate (CFAR) detector;
- adaptive implementation of optimum detectors when the clutter covariance matrix is unknown, the adaptive matched filter (AMF) and the Kelly's receiver;
- covariance matrix estimation: the sample covariance matrix (SCM) estimate;
- performance analysis and design trade-offs.

4.3. Optimum and adaptive radar detection of targets in heavy-tailed non-Gaussian clutter

Coherent radar target detection against heavy-tailed non-Gaussian clutter is the subject of the third part of the course. In high resolution radar systems the disturbance cannot be modelled as Gaussian distributed and the classical detectors suffer from high losses. Then, according to the adopted disturbance model, optimum and sub-optimum detectors are derived and their performance analyzed against a non-Gaussian background. First, it is solved the problem of designing the optimum algorithm for detecting a completely known radar signal or with unknown amplitude and initial phase against disturbance which is modelled as correlated non-Gaussian clutter modelled as a compound-Gaussian process. The performances are evaluated and compared to those of the classical matched filter detector. The problem of detecting fluctuating random signals, possibly with unknown parameters, against correlated compound-Gaussian clutter is then described exploiting different degrees of knowledge on target and clutter statistical characteristics. A generalized likelihood ratio test (GLRT) detector and a fully adaptive CFAR detector are derived and different novel interpretations of the detection algorithms were provided in order to highlight the
relationships and the differences among them and the links with the Gaussian clutter case. The CFAR detector is then obtained following different approaches, e.g. by approximating in a linear fashion the optimal nonlinear data-dependent threshold (DDT) of the GLRT detector.

Modern radar systems generally operate in non-homogeneous and non-stationary clutter environment. In this condition the amplitude statistics and the power spectral density (PSD) of the disturbance are unknown. Therefore, the adaptive versions of the algorithms previously described, which estimate the clutter covariance matrix, are then introduced. An estimation algorithm is described which guarantees the CFAR property and its performance are investigated and compared to that of the maximum likelihood (ML) estimator. The important cases of target signal partially unknown or modelled as a subspace random process are also analyzed. The problem of detecting subspace random signals against correlated non-Gaussian clutter exploiting different degrees of knowledge on target and clutter statistical characteristics is then addressed. The optimum Neyman-Pearson (NP) detector, the generalized likelihood ratio test (GLRT), and a constant false alarm rate (CFAR) detector are sequentially derived both for the Gaussian and the compound-Gaussian scenarios. Different interpretations of the various detectors are provided to highlight the relationships and the differences among them. In particular, it is shown how the GLRT detector may be recast into the form of the generalized whitening matched filter (GWMF), which is the GLRT detector against Gaussian disturbance, compared to a data-dependent threshold.

The performance of the various detectors against both simulated data and measured high resolution sea clutter data are illustrated as a function of the various clutter and signal parameters. Numerical examples concern a Space-Time Adaptive Processing (STAP) scenario and a ground-based surveillance radar system scenario.

In this part of the course, the following topics will be covered:

- Structure and interpretations of the optimum coherent detector in compound-Gaussian clutter: (i) the likelihood ratio test; (ii) the estimator-correlator; (iii) the whitening matched filter and data-dependent threshold (DDT);
- suboptimum detection in compound-Gaussian clutter based on the three interpretations of the optimum detector;
- adaptive implementation of optimum detectors when the clutter covariance matrix is unknown;
- covariance matrix estimation: the normalized sample covariance matrix (NSCM) estimate, the approximate maximum likelihood (AML) estimate;
- performance analysis, design trade-offs.
Implementation of radar signal processing algorithms. A practical session will follow after each main section. The practical session will consist of programming and running MATLAB codes that will implement simple radar signal processing algorithms. The practical session will help the understanding of concepts and techniques.

5. Learning outcomes
Having successfully completed this course, students should:

- understand the coherent radar array data model and its statistical analysis;
- understand the optimal and adaptive coherent detection of radar targets problem;
- know the techniques and algorithms that are currently used and choose which ones are the most suitable for a given scenario;
- understand the significance of disturbance modelling and analysis in a radar system;
- be able to analyze real clutter data;
- be able to generate synthetic data for radar system performance simulation
- be able to implement algorithms for radar target detection
- be able to analyze radar detection algorithm performance by the Monte Carlo method;
- be able to understand how the target signal model affects the structure of the detectors and its performance.

6. Textbook
Detailed presentation slides will be made available to students before the course starts and the Matlab code to solve the proposed problems will be made available during the course.

7. Description of topics (L=lecture, P=Practical session)

L.1. Introduction to the radar detection problem (1h)
   L1. Problem definition and nomenclature

L.2. Array Radar Signal Model (3h)
   L.2.1. Radar data cube.
   L.2.2. Complex envelope of a narrowband signal.
   L.2.3. The binary hypothesis testing problem.
   L.2.4. Noise space-time model.
   L.2.5. Target space-time model.
       L.2.5.1. Doppler frequency.
       L.2.5.2. Swerling target models.
L.2.5.3. Ambiguity function.
L.2.6. Jamming space-time model.

L.3. Radar clutter space-time model (2h)
L.3.2. Theoretical and empirical model for clutter reflectivity.
L.3.3. The compound-Gaussian model.
L.3.4. Spectral models for sea and land clutter.

L.4. Radar clutter analysis and simulation (3h)
L.4.1. Sea clutter statistical analysis.
L.4.2. Sea clutter non-stationarity.
L.4.3. Land clutter statistical analysis.
L.4.4. Clutter simulation.

P.1. Radar clutter random generation and real clutter data analysis (3h)
P.1.1. Random generation of correlated Gaussian clutter data.
P.1.2. Amplitude analysis of selected sea/land clutter dataset.
P.1.3. Spectral analysis of selected sea/land clutter dataset.

L.5. Optimum coherent detection in Gaussian disturbance (7h)
L.5.1. The Neyman-Pearson criterion and the likelihood ratio test.
L.5.2. Target signal models for different degrees of a priori information.
L.5.3. Perfectly known target signal: the coherent whitening matched filter detector.
L.5.4. False alarm and detection probabilities of the coherent WMF detector.
L.5.5. Swerling I target signal: the noncoherent WMF.
L.5.6. Deterministic unknown complex amplitude: the GLRT.
L.5.7. False alarm and detection probabilities of the noncoherent WMF.
L.5.8. Estimator-Correlator interpretation of the WMF.
L.5.9. Other performance metrics: response patterns and sidelobe levels.
L.5.10. Optimal beamforming and classical beamforming.
L.5.11. Tapering of the space-time steering vector.
L.5.12. Other performance metrics: Improvement Factor (IF) and Array Gain (AG).
L.5.13. DOA, Doppler frequency and complex amplitude maximum likelihood estimation.
L.5.15. Subspace Gaussian target: the subspace matched detector (MSD) and its estimator-correlator interpretation, the energy detector (ED).

P.2. Implementation and performance analysis of detectors in Gaussian clutter (3h)
   P.2.1. Doppler processing: Implementation of the non-coherent WMF, MF, ED, and NMF.
   P.2.2. Performance analysis for Swerling I and Swerling II target signals.
   P.2.3. Spatial processing: performance analysis of Capon beamformer and classical beamformer for the DOA estimation of Swerling II target signals in the presence of correlated clutter and multiple wideband jammers.

L.6. Adaptive detection in Gaussian noise (2h)
   L.6.1. CFAR detection: the CFAR MSD, the normalized matched filter (NMF), the adaptive NMF (ANMF), the Adaptive Coherence Estimator (ACE).
   L.6.4. Detection of mismatched targets and the Adaptive Subspace Detector (ASD).

L.7. Optimum coherent detection of targets in heavy-tailed compound-Gaussian clutter (5h)
   L.7.2. The estimator-correlator and the whitening matched filter compared to a data dependent threshold interpretations.
   L.7.3. Suboptimum detection in compound-Gaussian clutter based on the three interpretations of the optimum detector.
   L.7.4. Performance analysis and design trade-offs.
   L.7.5. The GLRT detector in compound-Gaussian clutter with Inverse-Gamma texture: the Linear Threshold Detector (GLRT-LTD).

P.3. Implementation and performance analysis of detectors in Gaussian clutter (3h)
   P.3.1. Doppler processing: Implementation of optimum detector for Swerling I target signal in compound-Gaussian clutter with Gamma texture and inverse Gamma texture, comparison with the WMF and with the NMF detectors.
   P.3.2. Performance analysis for Swerling I target signal in compound-Gaussian clutter with Gamma texture (K-distributed clutter) with a priori known covariance matrix.
P.3.3. Performance analysis for Swerling I target signal in compound-Gaussian clutter with inverse Gamma texture (generalized Pareto distributed clutter) with a priori known covariance matrix.

L.8. Adaptive coherent detection of targets in heavy-tailed compound-Gaussian clutter (3h)
   L.8.1. Adaptive implementation in compound-Gaussian clutter when the clutter covariance matrix is unknown.
   L.8.2. The sample covariance matrix (SCM) method, the normalized SMC, and the (approximate) ML covariance matrix estimate.
   L.8.3. Adaptive implementation in compound-Gaussian clutter when the target signal is 1-D unknown (unknown steering vector).

P.4. Implementation and performance analysis of adaptive detectors in compound-Gaussian clutter (3h)
   P.4.1. Doppler processing: Implementation of the GLRT detector for compound-Gaussian clutter, Kelly's GLRT and ANMF.
   P.4.2. Performance analysis for Swerling I target signal in compound-Gaussian clutter with Gamma texture and inverse Gamma texture.
   P.4.3. Performance analysis by processing real sea clutter data.
8. Lecture programme

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<th>Time</th>
<th>Mon 25/7</th>
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Fulvio Gini: **black**

Maria S. Greco: **blue**

9. Course Load

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